M87 Image at 43 GHz with “Sparse Modeling”

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Our Motivation

“Getting Black Hole Image”

- **Development of a new imaging technique with super-resolution.**
- **Application of the new technique to observational data.**
- **How resolved image can we get?**

Imaging with the interferometer (I)

- **Basic Equation:** 2D Fourier Transform between the image and visibility

$$I_\nu(x,y) = \int\int dudv\, V_\nu(u,v) e^{-2\pi(ux+vy)}$$

- **Spatial frequency (u, v):** baseline vectors seen from the target source

- **What does interferometer observe?:**
  Fourier components at various baseline lengths (i.e. spatial frequencies)

- **How to Image:**
  In actual case, discrete Fourier transform of sampled visibility
  is performed to obtain images

Imaging with the interferometer

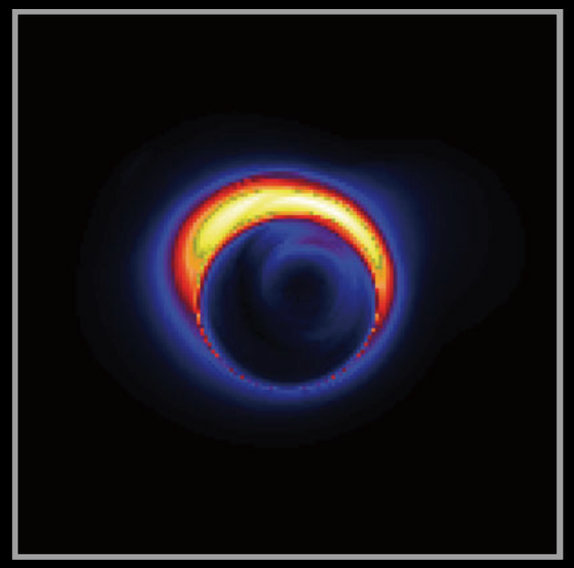
- In actual case：Imperfect sampling of Fourier components
  - 0-padding is used to obtain an image
    assuming visibilities of zero for unsampled Fourier components

- This cause finite resolutions and side lobes
  resolution: $\Theta \sim \lambda / B$ （λ：wavelength、B：baseline length）

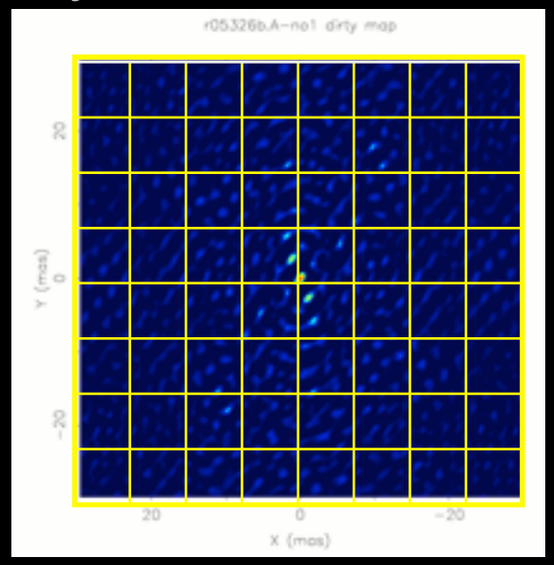
Traditional method (CLEAN = Matching Pursuit in Statistical Mathematics)

convolution (Dirty image)

Real



Synthesized beam



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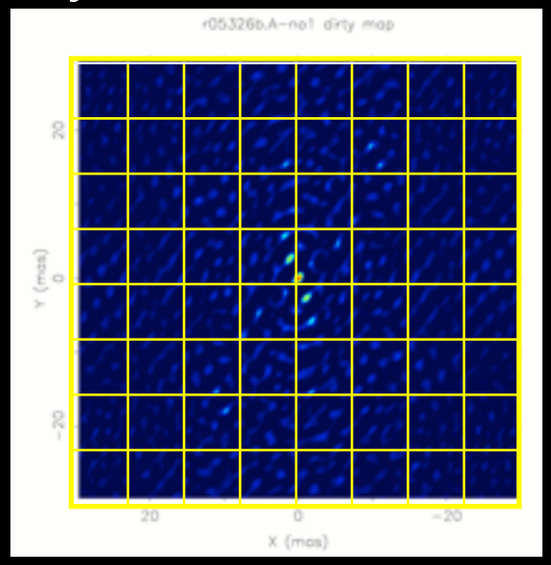
Synthesized beam



=>

Reconstructed Image:

consisting of a minimum number of point sources

reconstructing sparse images on the image plane

Sparse Modeling

Ill-posed problems

- Linear equations can be solved if number of equations M is larger than number of parameters N (i.e., requires $M > N$)

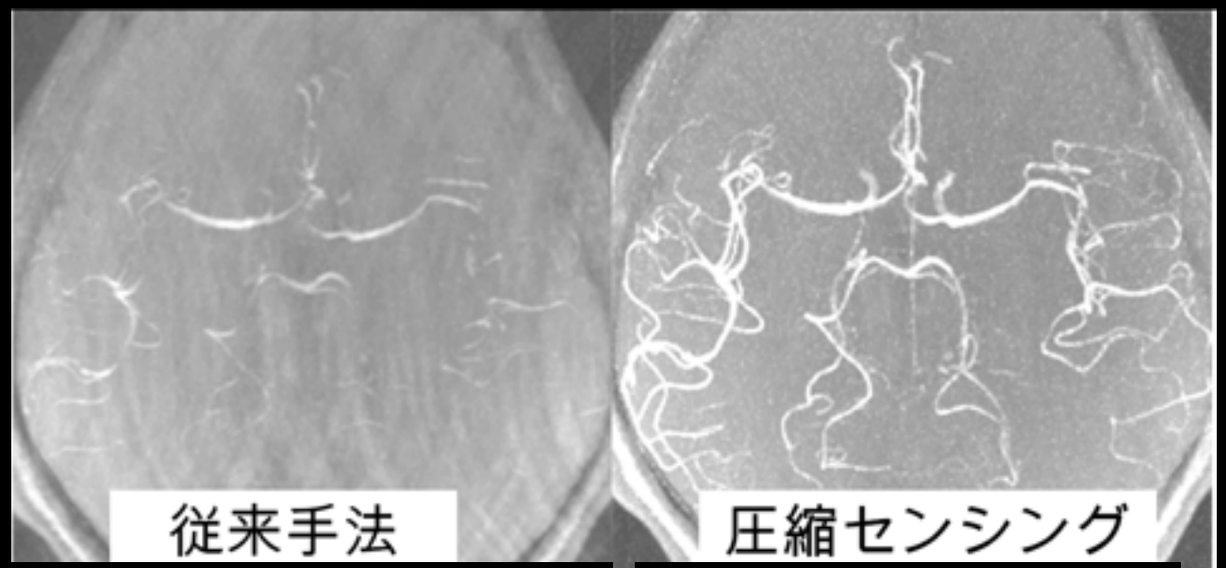
- Otherwise($M<N$), it becomes an ill-posed problem (can not be solved)

Idea of the sparse modeling to solve ill-posed problems

- If number of effective parameters (non-0 parameters) N' is smaller than M, equations can be solved (sparse solution)

- Mathematical background: (Donoho, Candes & Tao 2006; Compressive sensing)

- Compressing Sensing is now one of standard techniques for MRI (e.g. Lustig et al. 2008)



without Sparse Modeling with Sparse Modeling

MRI image of the cerebral blood vessel

Idea of Imaging with CLEAN (Matching Pursuit)

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ \cdots \\ \cdots \\ I_N \end{pmatrix} = \mathbf{A}^{-1} \times \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_M \\ 0 \\ 0 \end{pmatrix}$$

Actual Data

0 padding

- do 0 padding to equal numbers of data and image grids
- Try to find a sparse solution on the image plane

Idea of Imaging with Sparse Modeling

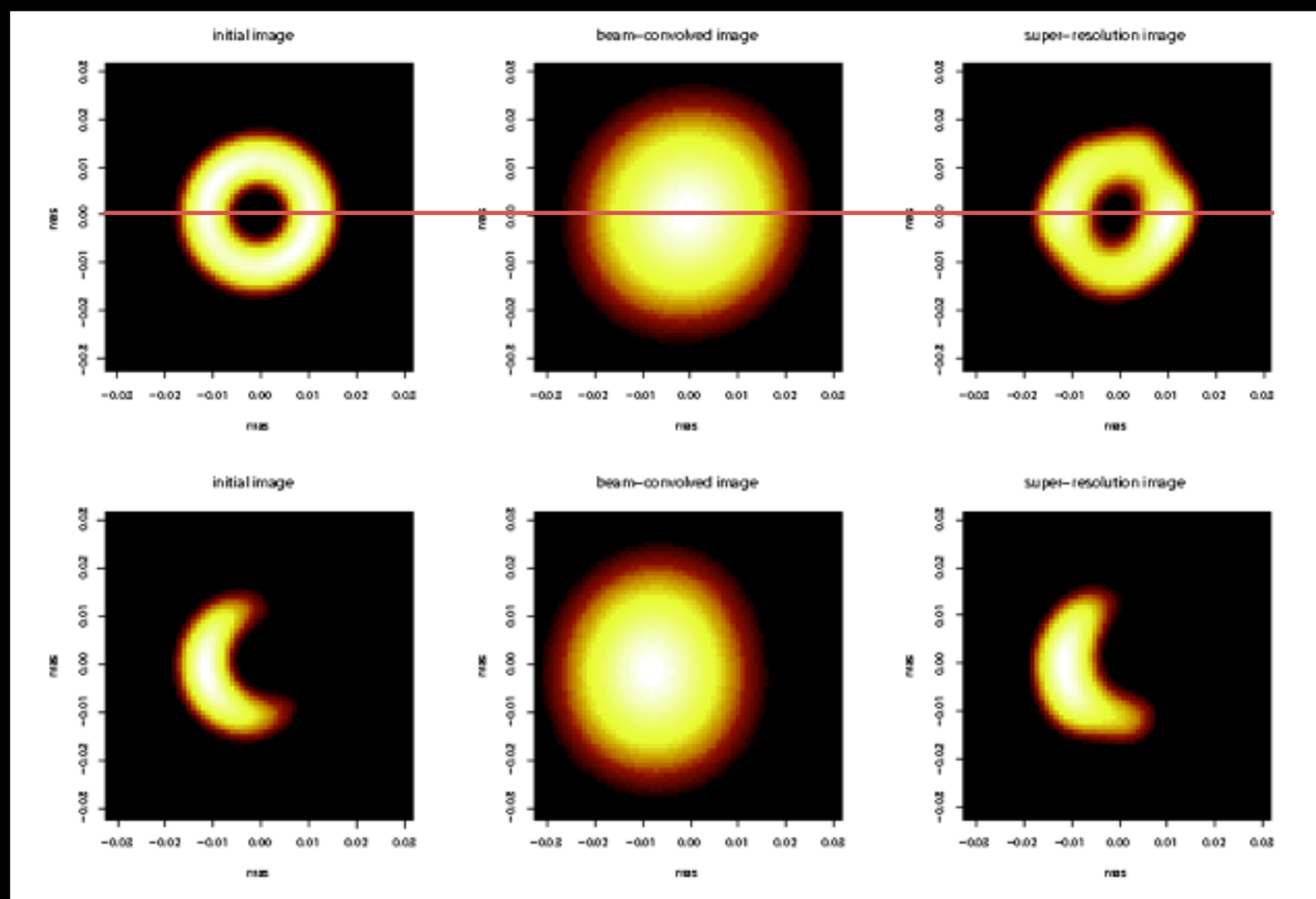
$$\underbrace{\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_M \end{pmatrix}}_{\text{Actual Data}} = \mathbf{A} \times \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ \cdots \\ \cdots \\ I_N \end{pmatrix}$$

grids with brightness of zero

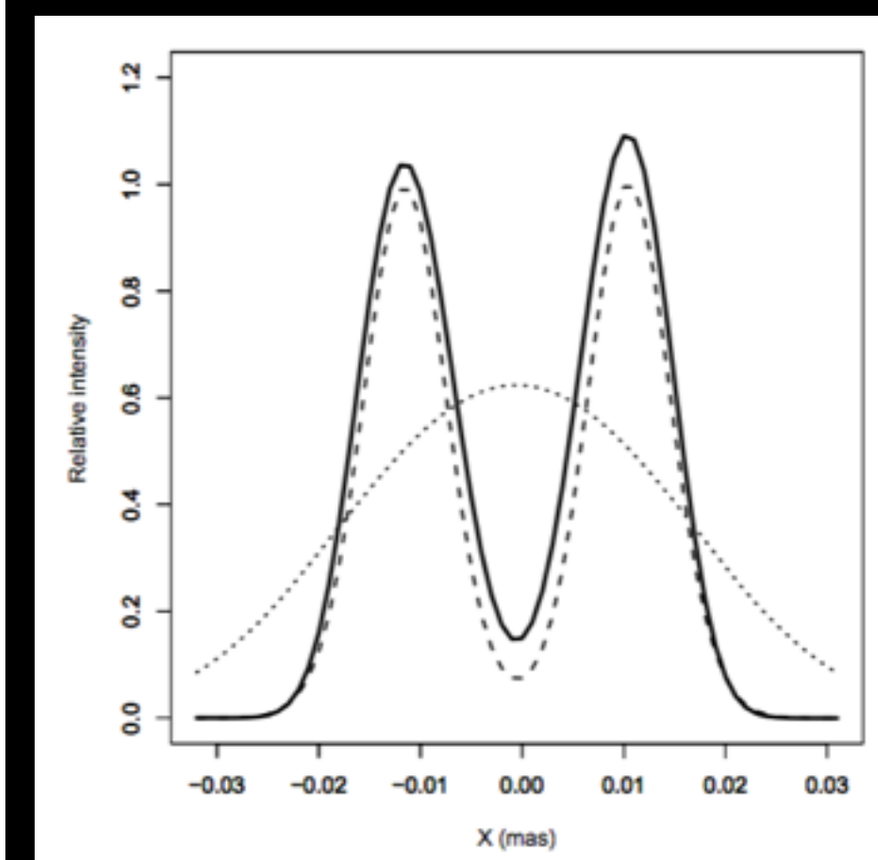
- ill-posed equations can be solved by focusing on "sparseness" of solutions.
- Try to find a sparse solution in the visibility plane
- Reconstructed image not affected by 0-padding
  → possibly we can get super-resolved image

Image Reconstruction with LASSO

Model Diffraction Limit LASSO image



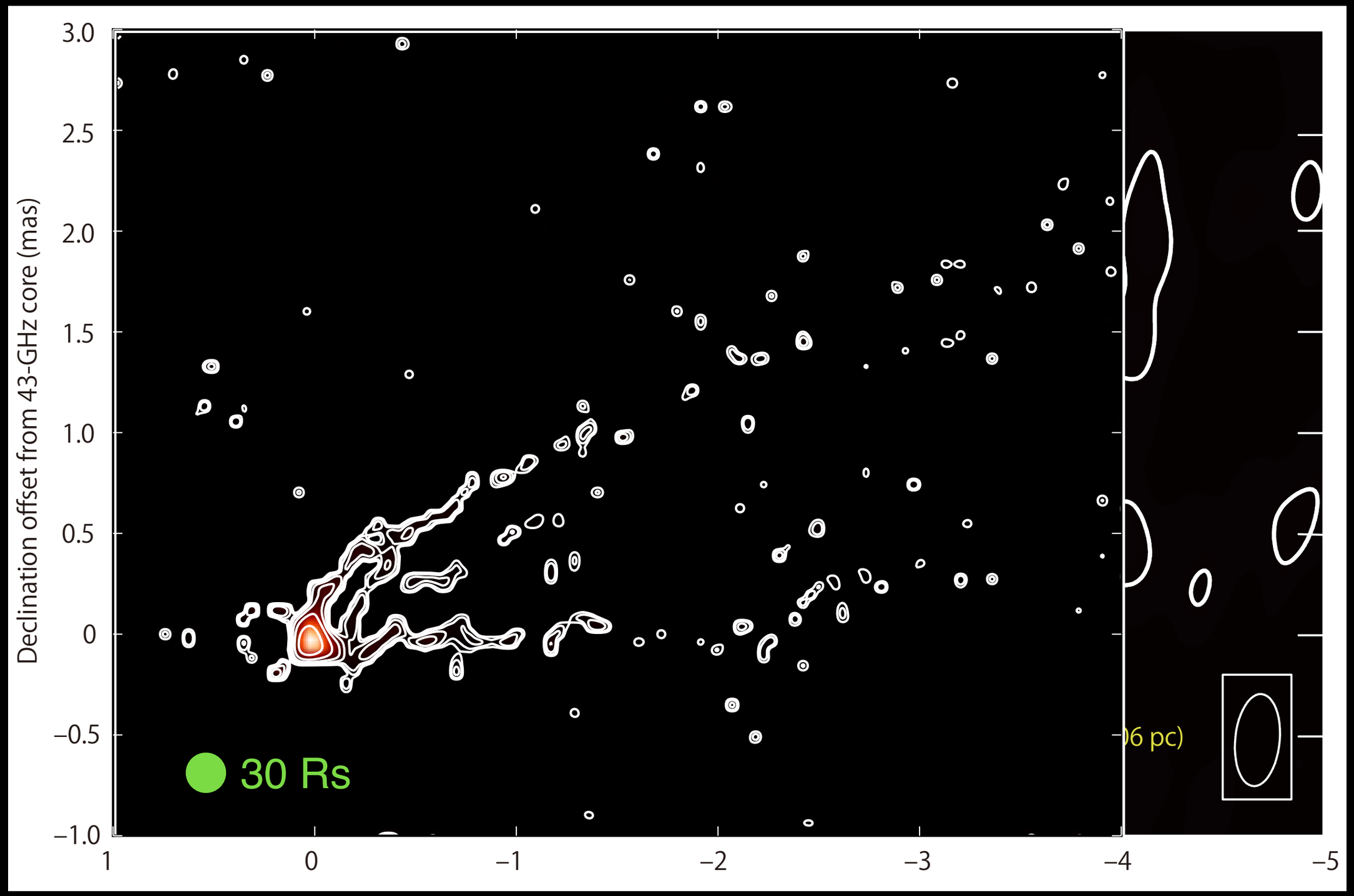
Sliced image



M87, Shadow Diameter ~ 20 uas (for the case of $M_{BH} = 3 \times 10^9\ M_{solar}$)

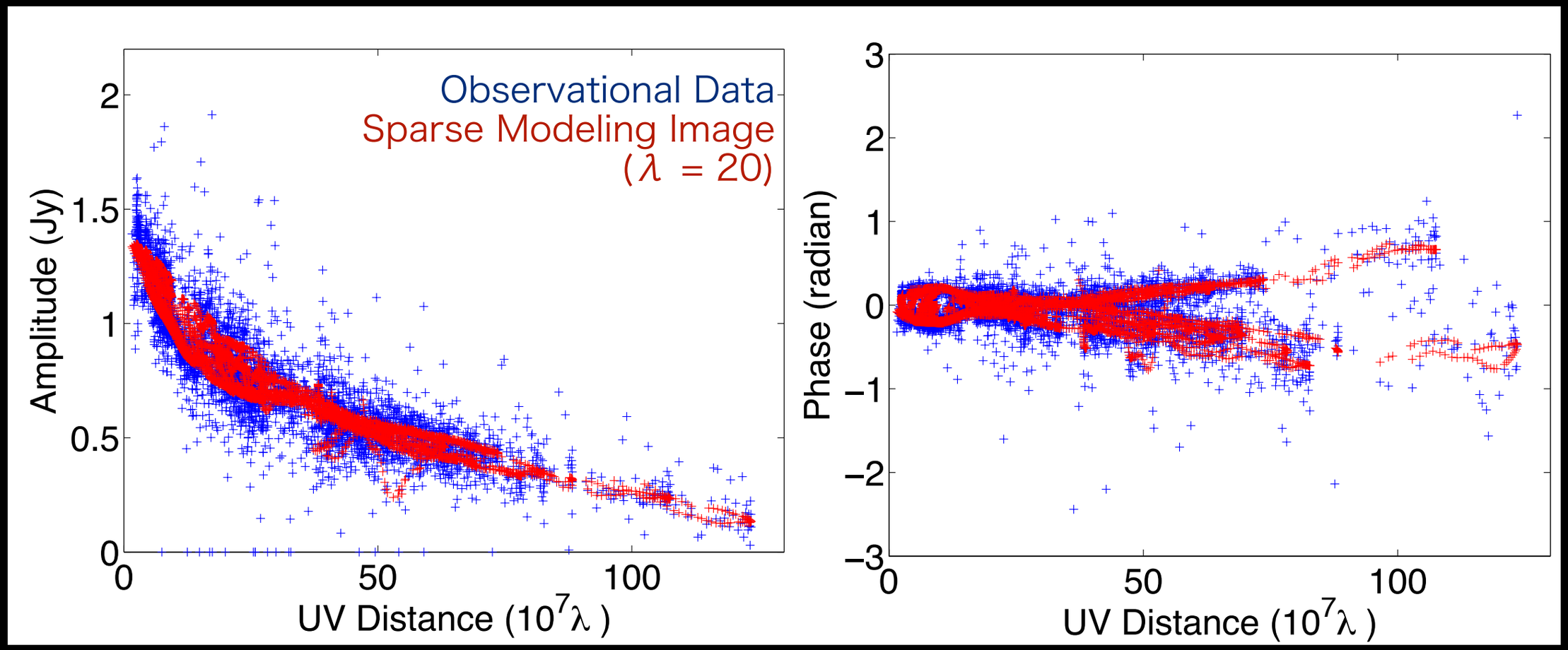
(Honma, Akiyama, Uemura & Ikeda 2014, PASJ)

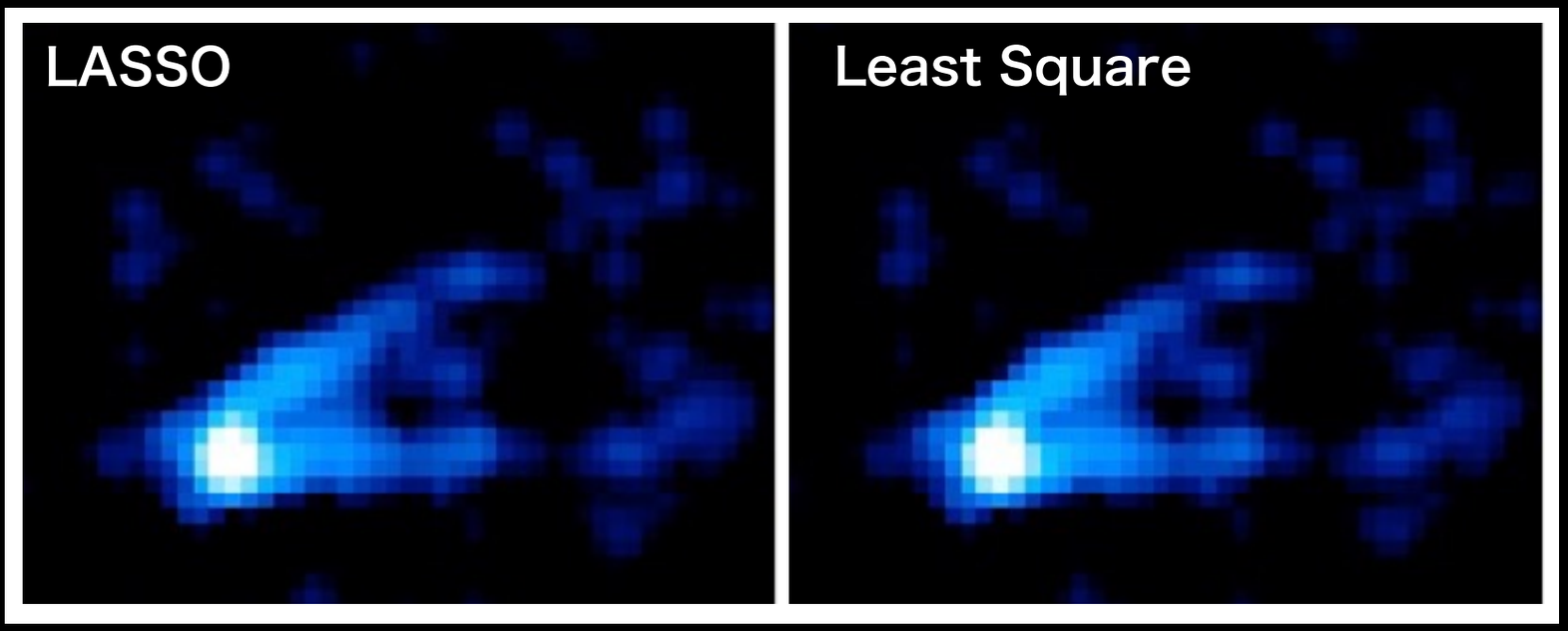
Application to M87 43GHz Data Observed with VLBA



Hada et al. 2011, Nature (43 GHz/7 mm)

Model visibility vs data





Visibility Sample: 4472

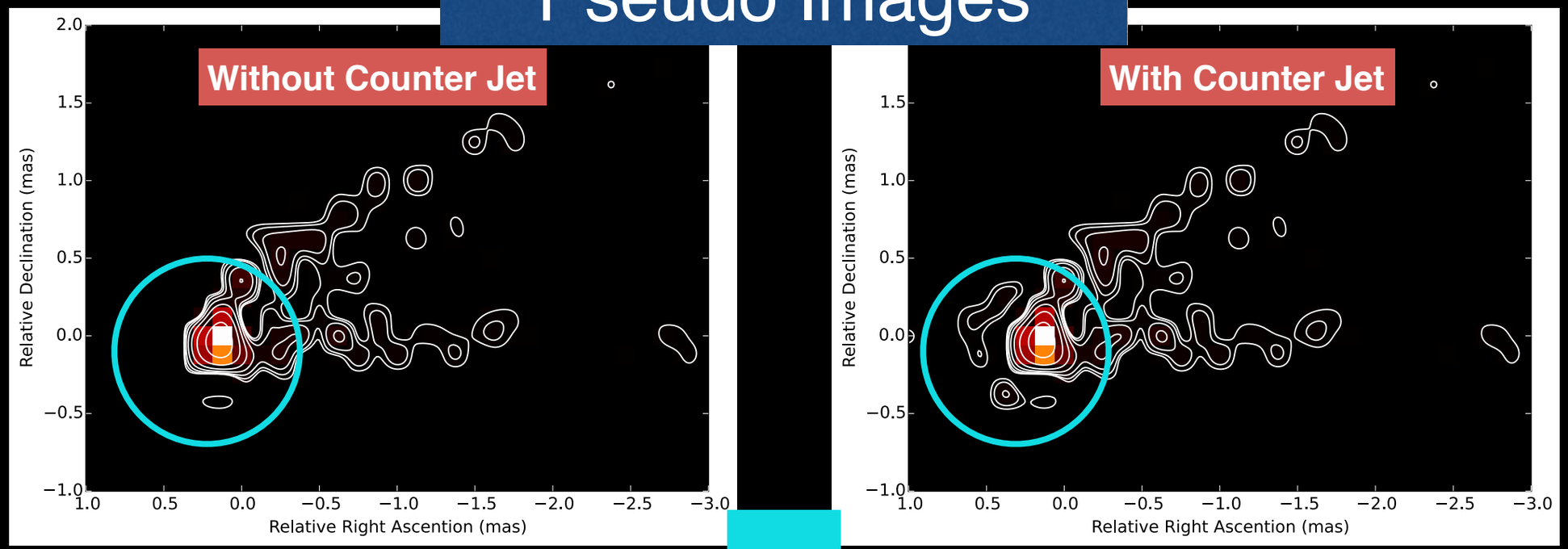
↓ **Well-posed problem**

Image pixel: $64^2 = 4096$

In well-posed program, LASSO and Least Square provides absolutely same solution

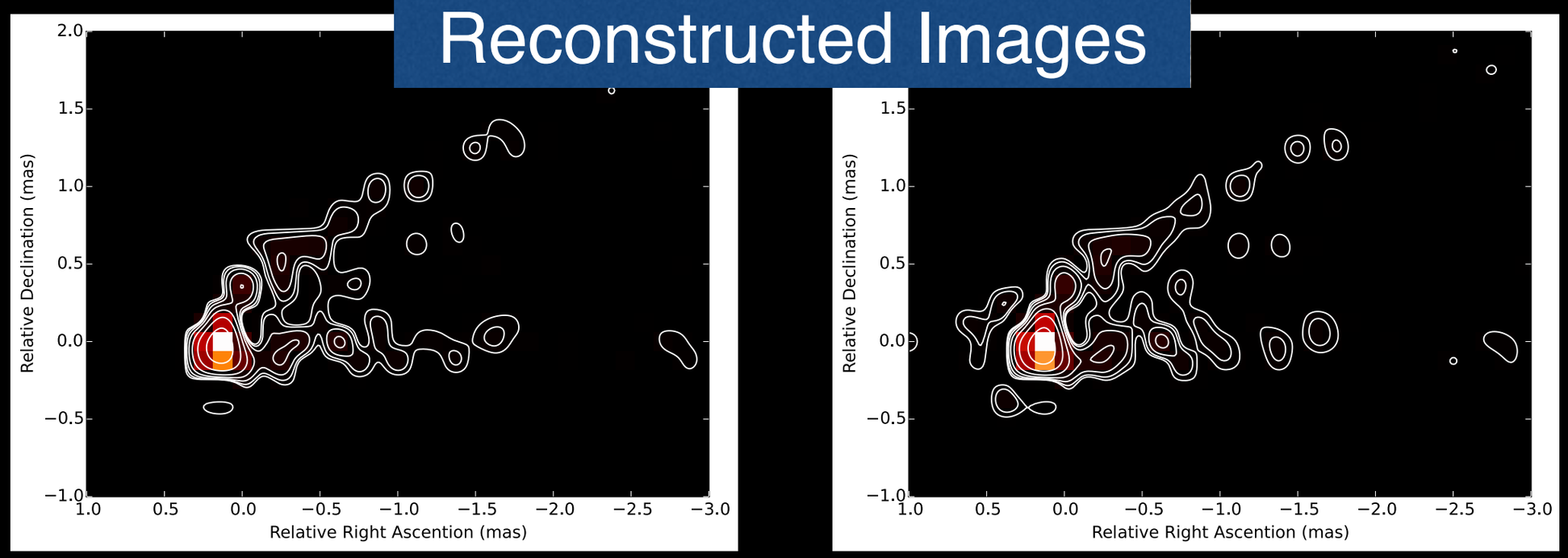
Counter-jet feature is real?

Pseudo Images

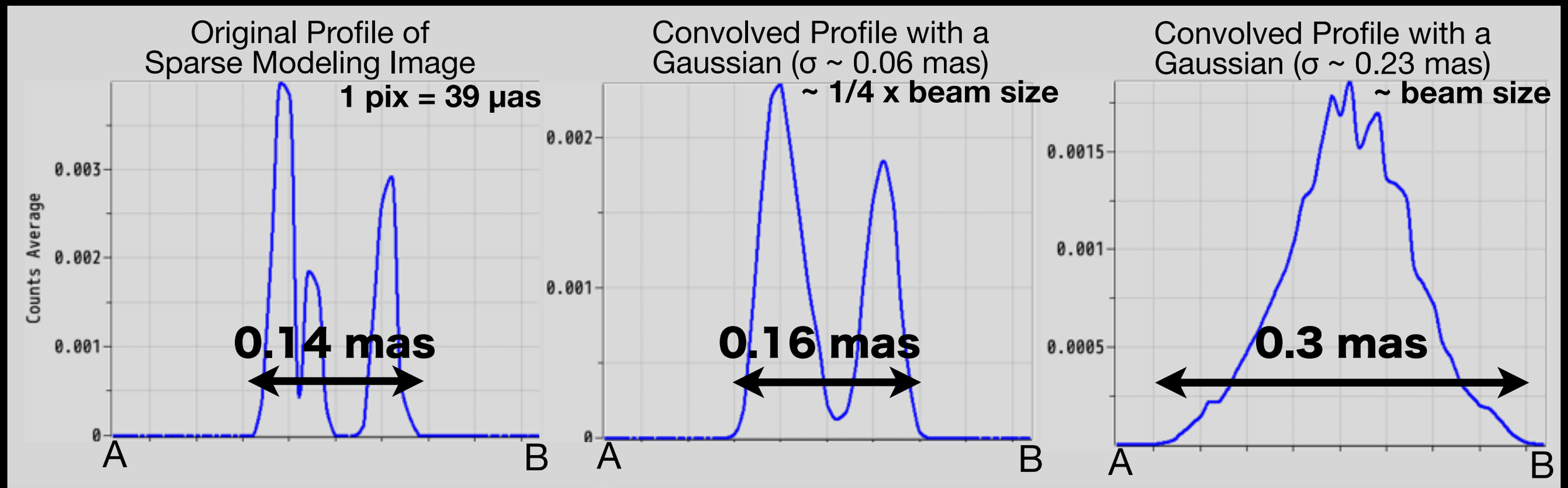


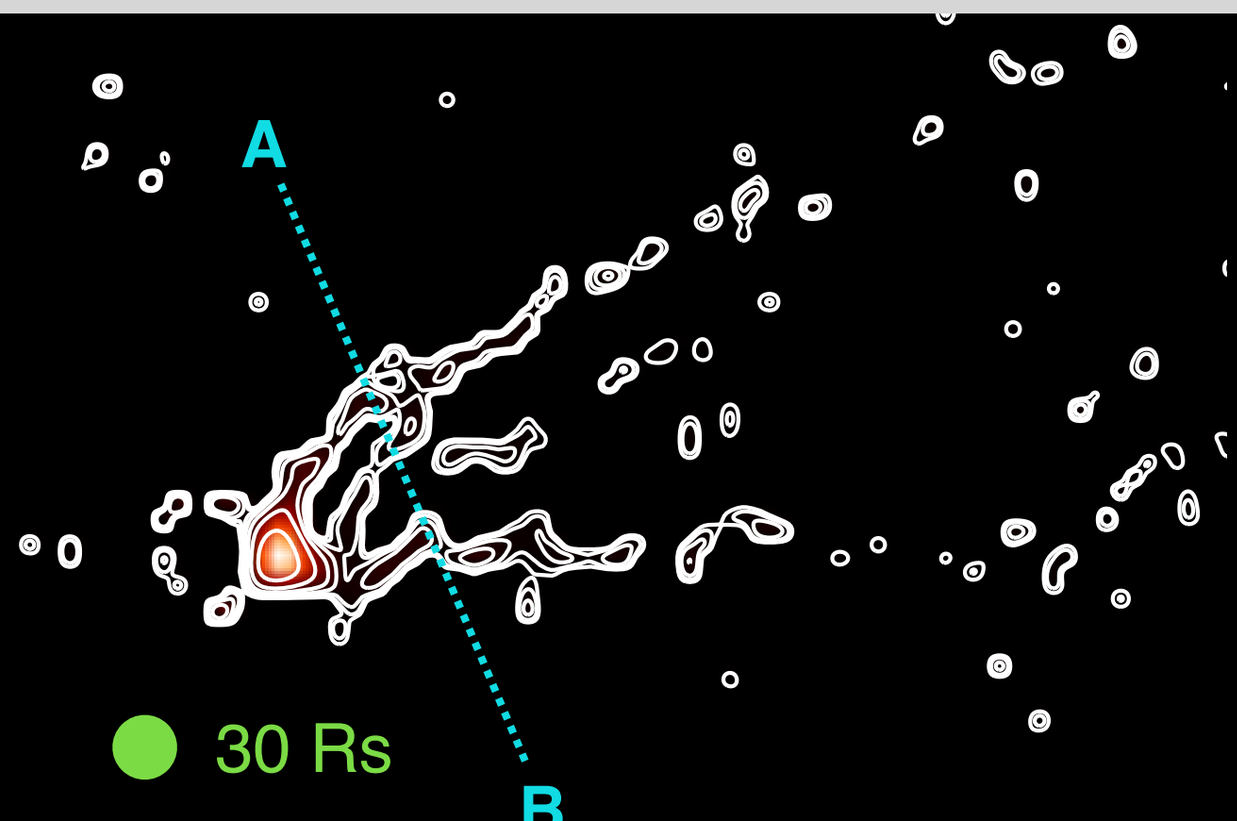
Create Visibilities, Add noise
and creating image again with LASSO

Reconstructed Images



Limb Profile of Sparse Modeling Image





- Limb-brightening structure is produced closer to the core than known with the previous image with CLEAN.

Summary

- We successfully obtain the M87 43GHz image with “Sparse Modeling.”
- Limb-brightening structure can be seen in our image, which is consistent with the CLEAN image.
- However, the structure is produced closer to the core.
- Our new imaging technique enables us to get super-resolved image.