Sparse Modeling for Radio Interferometry
— Basics, Applications and its Current Status —

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On behalf of Many On-going Research Projects related to Sparse Modeling

Developer Team: Shiro Ikeda (ISM), Fumie Tazaki (NAOJ), Kazuki Kuramochi (U Tokyo), Mahito Sasada (NAOJ), Mareki Honma (NAOJ), CASA Developer Team (Nakazato et al., NAOJ)

Radio Interferometry: Sampling Fourier Components of the Images

Sampling is NOT perfect

(Images: adapted from Akiyama et al. 2015, ApJ; Movie: Laura Vertatschitsch)
Interferometry Imaging: Observational equation is *ill-posed*

\[
Y \quad = \quad A \cdot X
\]

- Sampling is NOT perfect
  \[M < N\]
- Equation is *ill-posed*: infinite numbers of solutions
- Interferometric Imaging:
  Picking a reasonable solution based on a prior assumption

\[
Y = \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_M
\end{bmatrix}
= \begin{bmatrix}
\exp(i2\pi u_1 x_1) & \exp(i2\pi u_1 x_2) & \cdots & \exp(i2\pi u_1 x_N) \\
\exp(i2\pi u_2 x_1) & \exp(i2\pi u_2 x_2) & \cdots & \exp(i2\pi u_2 x_N) \\
\exp(i2\pi u_3 x_1) & \exp(i2\pi u_3 x_2) & \cdots & \exp(i2\pi u_3 x_N) \\
\vdots & \vdots & \ddots & \vdots \\
\exp(i2\pi u_M x_1) & \exp(i2\pi u_M x_2) & \cdots & \exp(i2\pi u_M x_N)
\end{bmatrix}
\cdot
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_N
\end{bmatrix}
\]
Philosophy: Reconstructing images with the smallest number of point sources within a given residual error

Computationally very expensive!!
(It can be solved for $N < \sim 100$)

- $L_0$ norm is not continuous, nondifferentiable
- Combinational Optimization

$\|X\|_0 = \text{number of non-zero pixels in the image}$
Sparse Reconstruction: CLEAN (greedy approach)

CLEAN (Hobgom 1974) = Matching Pursuit (Mallet & Zhang 1993)

Computationally very cheap, but highly affected by the Point Spread Function

Dirty map: FT of zero-filled Visibility

Point Spread Function: Dirty map for the point source

Solution: Point sources + Residual Map

(3C 273, VLBA-MOJAVE data at 15 GHz)
**Sparse Reconstruction: CLEAN (greedy approach)**

**CLEAN** (Hobgom 1974) = Matching Pursuit (Mallet & Zhang 1993)

CLEAN is problematic for the black hole shadows?

- Ground Truth
- CLEAN

<table>
<thead>
<tr>
<th>Ground Truth</th>
<th>CLEAN</th>
<th>CS-CLEAN</th>
</tr>
</thead>
</table>

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For the interferometer “clean” beam, and the reconstructions blurred with the clean beam (nominal) and the measured with the same fidelity as the model blurred to that resolution.

Choosing a restoring beam that is too small produces an image rapidly for restoring beams smaller than the optimal resolution, the MEM image fidelity is relatively una than from CLEAN for all values of restoring beam size. Most importantly, while the CLEAN curve NRMSE increases significantly smaller beam size than the CLEAN reconstruction, demonstrating MEM’s superior ability to superresolve dirty image residuals, as discussed at the end of Section 3.
Sparse Reconstruction: L1 Regularization

**LASSO (Tibshirani 1996)**

**Convex Relaxation:** Relaxing L0-norm to a convex, continuous, and differentiable function

\[
\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 < \varepsilon
\]

Equivalent:

\[
\min_{\mathbf{x}} \left(\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \Lambda_l \|\mathbf{x}\|_1 \right).
\]

Chi-square Regularization on sparsity

- Reconstruction purely in the visibility domain:
  Not affected by de-convolution beam (point spread function)
- Many applications after appearance of Compressed Sensing
  (Donoho, Candes+)

Kazu Akiyama, Mizusawa VLBI Observatory Users Meeting, NAOJ, Japan, 2017/11/03
Sparse Reconstruction: L1 Regularization

LASSO (Tibshirani 1996)

(Honma, Akiyama, Uemura & Ikeda 2014, PASJ)
Note: A “Popular” Wrong Statement

Imaging techniques can provide the perfect reconstruction if we have an infinite SNR (i.e. no noises) on data.

They can achieve even infinite angular resolutions in this case.

Interferometric Imaging:
- Regardless of noises, we have a infinite number of solutions fitting data
- It just picks a reasonable solution based on a prior assumption
  If the prior assumption is wrong, images can be wrong.

The angular resolution limit to distinguish 2 discrete sources from 1 source
~ $0.25 \frac{\lambda}{D}$ (no noises; Narayan & Nityananda 1986
with noises; Honma et al. 2014, PASJ and many other papers)
Pursuing only sparsity is not optimal

A key assumption in CLEAN and L1 regularization: images must be sparse.

May NOT work!
- Extended source
- Even compact source with too small image pixels

Akiyama et al. 2017b, AJ

We need somewhat sparse and smooth images NOT depending on adopted sizes of imaging pixels.
Sparse Modeling on the Gradient Image

\[ \min_x \left( \|y - Ax\|^2_2 + \Lambda_l \|x\|_1 + \Lambda_t \|x\|_{tv} \right) \]

- **Chisquare**: \( \|y - Ax\|^2_2 \)
- **L1 norm**: \( \|x\|_1 \)
- **Total Variation**: \( \|x\|_{tv} \)

Regularizing the **sparsity on the gradient domain**

= Favoring smooth images

\[ \|x\|_{tv} = \sum_i \sum_j \left( |x_{i+1,j} - x_{i,j}|^2 + |x_{i,j+1} - x_{i,j}|^2 \right) \]

Kuramochi et al. 2017
submitted to ApJ
### Application to Real Data: Protoplanetary Disk

**ALMA Observations of Protoplanetary Disk HD 142527 (345 GHz)**

**Compact configuration**

<table>
<thead>
<tr>
<th>Nominal Resolution</th>
<th>Superresolution (same to the intermediate configuration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAN (Cyc3)</td>
<td>CLEAN (Cyc3)</td>
</tr>
<tr>
<td></td>
<td>Sparse Modeling (Cyc3)</td>
</tr>
</tbody>
</table>


**Intermediate config.**

<table>
<thead>
<tr>
<th>Nominal Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAN (Cyc2)</td>
</tr>
</tbody>
</table>

Fukagawa et al. in prep.  
(Yamaguchi, Akiyama, & Kataoka et al. in prep.)
Applications to SKA Science: Faraday Tomography

EVPA rotation of radio waves in magnetized plasma

\[ \chi = \chi_0 + RM\lambda^2 \]

\[ RM \ (\text{rad m}^{-2}) \approx 811.9 \int \left( \frac{n_e}{\text{cm}^{-2}} \right) \left( \frac{B_{||}}{\mu\text{G}} \right) \left( \frac{dr}{\text{kpc}} \right) \]

Rotation angle is proportional to \( \lambda^2 \)

= phase rotation in linear Pol spectrum

This is very similar to what we usually see in interferometric data.

(e.g.) A point source in the image causes a phase rotation in the visibility, which is a spatial spectrum of the image.

\[ \Delta \varphi = 2\pi \chi_0 u \quad \text{for a point source at } x = x_0 \]

(x, u) for interferometric imaging; (RM, \( \lambda^2 \)) for Faraday Rotation
Applications to SKA Science: Faraday Tomography

\[ Y = A \times X \]

- **Y**: Linear Polarization Spectra
- **A**: Fourier Transform for Faraday depth
- **X**: Fourier Dispersion Function

**Black**: A galaxy model (from Ideguchi et al.)

**Red (doubled)**: Synthetic SKA observations

Faraday Dispersion Function

(Akiyama et al. in prep., Collaboration with SKA-JP Faraday Tomography WG)
EHT Imaging: Fusion of Young Powers & Divergence

Andre Young (SAO Astronomy)
Kazu Akiyama (MIT Astronomy)
Julian Rosen (UGA Mathematics)

Lindy Blackburn (SAO Astronomy)
Katie Bouman (MIT Computer Vision)

Andrew Chael (Harvard Physics)

Marki Honma
NAOJ Astronomy

Shiro Ikeda
ISM Statistical Mathematics

Fumie Tazaki
NAOJ Astronomy

Kazuki Kuramochi
U. Tokyo Astronomy

Michael Johnson
(SAO Astronomy)
Other imaging techniques from the EHTC

Maximum Entropy Method (MEM)

CHIRP (Machine-learning)
Bouman et al. 2016

Model
Smooth Model
2016
2017

| 50 µas |

| 50 µas |

Kazu Akiyama, Mizusawa VLBI Observatory Users Meeting, NAOJ, Japan, 2017/11/03
Challenges for VLBI Imaging

No good phase calibrators!
We need to carefully CLEAN
so that images are reasonably smooth and sparse, and consistent with closure phases.

Solution: Imaging from Amplitudes + Closure Phases

Sparse Modeling: Akiyama et al. 2017a, Kuramochi et al. 2017
CHIRP: Bouman et al. 2016
Challenges for VLBI Imaging

No good amplitude calibrations!
We need to carefully CLEAN so that images are consistent with amplitude gains of ~10-30 %. ...., etc....

Solution: Full Closure Imaging (Cl. Amplitudes + Cl. Phase)

M87 Jet Model (Moscibrodzka+17)

EHT 2017/2018 Full Closure Imaging

Sparse Modeling: Akiyama et al. in prep.
MEM & CHIRP: Chael et al. in prep.
Challenges for VLBI Imaging

Sgr A* (and M87) has a time variability.

Solution: regularize and solve movies.
(extension of sparse and other regularizers in time direction)

Simulation

EHT 2017+

00:00:00

Challenges for VLBI Imaging

Sgr A* is scattered!
Diffractive scattering: invertible
Refractive scattering: not invertible

Solution: regularize and solve the phase screen of the refractive scattering as well!

Unscattered
Scattered
Stochastic Optics Reconstructions

Summary

- Sparse Modeling and other EHT imaging techniques provide a new opportunity to obtain high-quality, high-resolution images (and movies) from various type of interferometric data sets.

- On-going wide application to various sources and other problems
  - Radio Stars, Protoplanetary disks, Jets
  - Faraday Tomography

- Softwares are under development and yet need a certain manpower for applications to real data, but with a huge potential of new sciences and publications !!

If you are willing to try algorithms for your projects, please visit us at MIT Haystack or NAOJ! We are happy to work with you!
Implementations

- **Sparselab (Akiyama et al.)**
  An open source imaging library by EHT-J

- **CASA Sakura Library (Nakazato et al.)**
  A FFT-based imaging function is under testing.

- **EHT imaging library (Chael et al.)**
  A general imaging & simulation library for the EHT
(Perhaps) no longer need the restoring beam

Ground Truth vs Convolved Images

Errors

NRMSE_image (%)

Sub-Keplerian

Ground truth

CS-CLEAN

ℓ₁ + isoTV

ℓ₁ + TSV

The restoring beam size


Kazu Akiyama, Mizusawa VLBI Observatory Users Meeting, NAOJ, Japan, 2017/11/03
Application to Real Data: VLBA M87 Data

**CLEAN (43 GHz)**

**Sparse Modeling (43 GHz)**

Clear reproduction of counter jets

Derived collimation profile of the M87 jet is consistent with 86 GHz data

(Tazaki et al., in prep.)